Antidumping Petition, Foreign Direct Investment, and Strategic Exports

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Abstract

I examine how the demand for protection (in the form of an antidumping duty petition) is affected by the foreign firm’s FDI opportunity. In equilibrium, the lobbying is either blockading, deterring, or accommodating FDI. When FDI is deterred, the lobbying decreases as the level of antidumping duty increases, and not only the home firm but also the foreign firm can benefit from an increase in the duty. This result leads to an interesting strategic export by the foreign firm, which I call protection-threat-defusing export. When the future duty depends on the foreign firm’s current export, the foreign firm may increase its export in order to dampen the home firm’s lobbying. Namely, the antidumping policy can induce more “dumping” by the foreign firm when it has an FDI opportunity.
1 Introduction

Decision of foreign exporting firms to make foreign direct investment (FDI) when they face protection in an importing country has been extensively discussed in the literature of international trade. One of the conventional arguments is that, when a tariff is imposed, foreign firms may set up subsidiaries in the importing country in order to avoid the tariff (tariff-jumping FDI). Various studies have elaborated the theory of tariff-jumping FDI, with the analysis of the optimal protection policy by the government of the importing country, and with the analysis of the strategic motive of investment by foreign firms. Empirically, Belderbos (1997) and Barrell and Pain (1998) studied how antidumping policies in the U.S. and Europe affect the flow of FDI from Japan, and found a positive relationship between antidumping protection and FDI. Blonigen (2000) examined the effect of the U.S. antidumping policy on FDI, using the data including non-Japanese firms as well as Japanese firms. At the firm level, he found that not many firms subject to the U.S. antidumping duties made tariff-jumping FDI.

Although these models of tariff-jumping FDI have examined the effects of protection policies on the decision of foreign firms to make FDI, there are not many studies that focus on the effect of FDI on the demand for protection by import-competing industries. In other words, it is the supply of protection by the government, not the demand for protection by import-competing industries, that is analyzed in the models of tariff-jumping FDI.

The objective of this paper is to analyze how the demand for protection is affected by the FDI opportunity of foreign firms. The bottom line of the idea in this paper is the one pointed out by Ellingsen and Warneryd (1999): import-competing firms do not want the maximal level of a tariff when the foreign firms can make foreign direct investment. In their paper, they assume that the government in the importing country is fully captured by the import-competing industry, so that the government sets the tariff to maximize the profit of the industry. In other words, the industry can costlessly demand the level of protection they want. Obviously, in such a setting, the tariff is always set at the level where the foreign firm is just short of making investment. On the other hand, in this paper we explicitly take account of the cost of protection seeking. Specifically, we model that protection seeking takes the form of a petition for antidumping duty (ADD).

When a petition for antidumping duty is filed, investigations are con-

\footnote{For example, see Brander and Spencer (1987), Smith (1987), Motta (1992), Haaland and Wooton (1995), and Ellingsen and Warneryd (1999).}
ducted by the International Trace Commission (ITC) and the Department of Commerce (DOC). The ITC determines if the import-competing industry is materially injured by the imports from foreign countries, while the DOC determines the existence of the alleged dumping and the size of the dumping margin, by observing the market prices in the foreign countries and in the U.S. There are two important observations of ADD protection relevant to the analysis in this paper. First, since the duty (or the tariff\textsuperscript{2}) is determined by technical calculations conducted by the DOC, typically import-competing firms (the ones who file a petition) have little influence on the determination of the duty when filing a petition. In empirical studies, it has been reported that the size of the duties determined by the DOC is not very much influenced by political pressure. On the other hand, it has been found that the injury determinations by the ITC is more subjective, and more susceptible to political pressure than the dumping determinations by the DOC.\textsuperscript{3} Second, the DOC’s existence-of-dumping test is easier to pass than the ITC’s injury test. Finger and Murry (1993) reported that only 10\% of the petitions has reached negative determination of the existence of dumping. Thus, whether the duty is imposed or not largely depends on injury determination by ITC.

From these observations, the following two crucial assumptions are made in our model. First, the size of the duty is dependent on the quantity exported by the foreign firm in a period before a petition is filed, but independent of the protection-seeking effort made by the import-competing firm. This assumption comes from the fact that DOC calculates the dumping margin by observing the market outcomes prior to the petition. The more export means the more dumping (i.e., the lower price of the product in the importing country), thus the higher the duty. Also, this assumption reflects that the import-competing industry has little influence on the determination of the duty when it files a petition. The second assumption is that the import-competing industry chooses how much to spend for a petition, and the spending for petition affects the probability of duty enforcement. The rationales of this assumption are that whether the duty is imposed or not largely depends on the ITC’s injury determination, and that the ITC’s injury determination is more susceptible to the political pressure than the DOC’s dumping determination. We assume that, the more the import-competing industry spends for a petition, the more convincing it can make the petition, and the more likely the duty will be enforced. In other words, protection-

\textsuperscript{2}I use the word “duty” and “tariff” interchangeably.

\textsuperscript{3}See Baldwin and Moore (1991), and Finger, Hall and Nelson (1982).
seeking effort of the import-competing industry is directed to increase the probability of protection.

For the analysis of ADD petition and tariff-jumping FDI, we consider a simple oligopolistic model where the single firm in an import-competing industry (from now on, we refer it as “the home firm”) and the single foreign firm compete in the home country. The home firm spends its resources to increase the probability of protection so as to maximize its expected profit. After observing the protection-seeking effort of the home firm, then, the foreign firm decides whether to make FDI or not, before the uncertainty with respect to the enforcement of protection is resolved. Namely, the foreign firm has an opportunity to make anticipatory tariff-jumping FDI.\footnote{For an empirical study of anticipatory tariff-jumping FDI, see Blonigen and Feenstra (1997).} A rationale that the foreign firm is engaged in anticipatory tariff-jumping FDI is that it may take a quite long time to build a subsidiary plant and that the foreign firm may lose profit if it does not have a plant when the protection is enforced. Also, we note that the time length of an ADD investigation may be long enough for the foreign firms to make FDI: a typical ADD case takes 280 to 420 days to reach final determinations.

From this setting of the model, we derive several interesting results. The first finding of the paper is that the level of protection-seeking effort chosen by the home firm is either blockading FDI, deterring FDI, or accommodating FDI, depending on the parameter values. In general, FDI is blockaded (deterred) when the foreign firm’s gain from investment is small (large) relative to the cost of investment, and the home firm’s gain from protection seeking is small (large) relative to the cost of the protection seeking. A numerical analysis shows that accommodation of FDI occurs only in a very limited range of parameter values. This suggests that the actual anticipatory tariff-jumping FDI may not be observed very much under ADD protection. This finding is consistent with the empirical finding of Blonigen (2000) mentioned above. Also, this finding provides an alternative answer to the question why intensive protection-seeking activities, in the form of antidumping petition, are not observed in all industries. As well known, the conventional answer to this question is that there is a free-riding problem in collective actions (Olson (1965)). Our model, on the other hand, gives an explanation without such an externality problem. When the foreign firm has an investment opportunity after the protection seeking of the home firm, the optimal level of protection seeking chosen by the home firm is often the one that deters FDI, which is below the level that would be chosen if the foreign firm were
not able to make FDI. In other words, the mere existence of an FDI opportunity, not an actual investment, can restrain the protection-seeking effort of the home firm.

The second result of the paper is in terms of comparative statics for the protection-seeking effort. When the optimal level of the effort is to deter FDI, the protection-seeking effort is decreasing in the tariff. An increase in the tariff makes the foreign firm engaged in FDI at the smaller level of protection probability. Knowing this, the home firm, whose profit is decreased when the foreign firm makes FDI, will lower its protection-seeking effort and thus lower the probability of protection to keep the foreign firm from making FDI. An important implication of this finding is that the foreign firm can benefit from an increase in the tariff, through a decrease in the probability of protection. In turn, this implication leads to an interesting strategic behavior of the foreign firm in a period prior to the protection-seeking stage, since the tariff depends on how much the foreign firm exported in a period prior to the protection-seeking stage.

An idea that the level of protection may depend on the market outcome in a period before protection enforcement is first examined by Bhagwati and Srinivasan (1976), in which a probability of quota enforcement in future depends on the current level of imports. The central result of their paper is that the exporting country reduces its exports when facing the future protection. Fischer (1992) and Reitzes (1993) studied endogenous protection in oligopolistic models, and derived the similar result that foreign firms strategically decrease the exports in order to lower the level of protection in future. On the other hand, Anderson (1992, 1993) showed that the possibility of protection in future may increase the current exports. When the exporting firms are facing the prospect of voluntary export restraint (VER), each firm will strategically increase its export in order to receive a larger share of export licences in future.

In our model, when the foreign firm facing endogenous protection, the direction of strategic change in its export is not uniform. On one hand, when the optimal protection-seeking effort chosen by the home firm is to blockade FDI, the foreign firm strategically decreases its export in order to lower the tariff. This is a conventional result obtained in most of endogenous protection literature. On the other hand, when the home firm is going to deter FDI, the foreign firm strategically increases its export in order to reduce the protection-seeking effort of the home firm. This is rather a new result. We name such a strategic increase in export “protection-threat-defusing export”. Having an FDI opportunity after the protection seeking of the home firm, the foreign firm can defuse the threat of protection by
increasing the future tariff and thus by making itself ready to invest at the smaller level of protection probability.

A recent paper by Blonigen and Ohno (1998) presented the similar results that the strategic reactions of the exporting firms facing the prospect of protection may not be uniform when the exporting firms have FDI opportunity. In their model, two exporting firms compete in the third-county market, each facing a firm-specific tariff which depends not only on its export levels but also on the export levels of its rival. Blonigen-Ohno showed that there is an equilibrium in which one firm whose cost of FDI is low strategically increases its current export and the other firm whose cost of FDI is high strategically decreases its current export. Despite the similarity of the results, however, the incentive for the strategic increase in the export and empirical implications are quite different between Blonigen-Ohno’s model and ours. In Blonigen-Ohno, an exporting firm may strategically increase its export in order to raise the future tariff on its rival’s export, while in our model, the foreign firm may strategically increase its export in order to influence the protection seeking effort of the home firm. Thus, the result of Blonigen-Ohno implies that when some firms strategically increase their exports, there must be other firms which decrease their exports, and that the increase in the exports must be followed by actual investment. Contrary, in our model all exporting firms may be engaged in a strategic increase in the exports. In addition, our model predicts that the strategic increases in the exports may not be followed by actual investment, since it is the existence of investment opportunity, not necessarily an actual investment, that affects the protection-seeking effort of the home firm. These differences in empirical implications provide testable hypothesis that distinguishes our model from Blonigen-Ohno’s.

The analysis in this paper can be contrasted to the theory of quid pro quo investment, initiated by Bhagwati et al., (1987), and developed by Dinopoulos (1989), Wong (1989), and Grossman and Helpman (1996) among others. In Wong (1989), foreign firms may be engaged in FDI before the labor unions in the home country lobby for protection, in order to appease the labor unions and thus to lower the threat of protection in future. In the quid pro quo theory, FDI prior to protection-seeking activities may be increased to defuse the threat of protection. On the other hand, in this paper the export prior to protection seeking activities may be increased to defuse the threat of protection.

The rest of the paper is organized as follows. In section 2.2, we describe the model. In section 2.3, the model is solved and the results are presented. Section 2.4 gives brief concluding remarks.
2 Description of the model

We consider a simple international duopoly model. There are two firms, one is the home firm and the other is the foreign firm, which compete in Cournot fashion. It is assumed that either there is no demand in the foreign country, or the market in the foreign country is completely protected, thus the competition between the home firm and the foreign firm takes place only in the home country.

The inverse demand function in the home country is given by \( p(q_h + q_f) \), with \( p' < 0 \). The variable \( q_h \) denotes the quantity sold by the home firm and \( q_f \) denotes the quantity sold by the foreign firm. If the foreign firm does not invest in the home country, \( q_f \) is provided only from exports. If the foreign firm makes FDI, then it can produce at its subsidiary in the home country, so \( q_f \) can be provided from the subsidiary as well as from export. The foreign firm’s export is subject to the per unit tariff, \( t \), if the protection is enforced in the home country, while the subsidiary production is not subject to the tariff.

For analytical simplicity, we assume that the firms have constant marginal cost of production: \( c_h \) denotes the marginal cost of the home firm, \( c_f \) denotes the marginal cost of the foreign firm’s production in the foreign country (possibly including the cost of transportation to export), and \( c_s \) denotes the marginal cost of the foreign firm’s production at the subsidiary. In order to focus on the role of FDI as tariff jumping, we assume that \( c_f < c_s < c_f + \bar{t} \), where \( \bar{t} \) is the prohibitive tariff. Namely, the foreign firm has no incentive to invest if there is no tariff, but it prefers to produce at the subsidiary if the tariff is high enough. Because of the assumption of constant marginal costs, the foreign firm supplies \( q_f \) either by export or by subsidiary production. More specifically, the foreign firm supplies \( q_f \) only from export (and no subsidiary production) if \( c_f + t \leq c_s \), and it supplies \( q_f \) only from the subsidiary (and no export) if \( c_f + t > c_s \) and if it has set up the subsidiary.

Now, let us describe the timing of the game. At stage 0, the home firm and the foreign firm are engaged in Cournot competition in the home country, without any protection policy and without any subsidiary production of the foreign firm. The tariff that will be enforced in the later stage is determined as a function of the export by the foreign firm in stage 0. At stage 1, given the tariff, the home firm decides how much to spend for a petition for antidumping duty. The more the home firm spends its resources for the petition, the higher the probability of protection will be. The probability of protection is denoted by \( \theta \in [0, 1] \). At stage 2, after observing the
protection seeking of the home firm (in the form of petition spending for ADD), the foreign firm decides whether to make FDI before the uncertainty about the protection enforcement is resolved (i.e., the investment is anticipatory tariff-jumping). The uncertainty about the protection enforcement is resolved after the stage 2 (and before stage 3). At stage 3, the home firm and the foreign firm again play the Cournot game in the home country. The export of the foreign firm is subject to the tariff.

3 Solving the model

Since the game is sequential, we solve it by backward induction. Let us start from analyzing stage 3.

3.1 Stage 3: Cournot competition

With probability $1 - \theta$, the tariff is not imposed. In this case, the home firm chooses $q_h$ to maximize

$$\pi_h(q_h, q_f) = p(q_h + q_f)q_h - c_h q_h$$  \hspace{1cm} (1)

and the foreign firm chooses $q_f$ to maximize\footnote{Since $c_f < c_s$, the foreign firm does not use its subsidiary when the tariff is not imposed, even if it has made FDI.}

$$\pi^*_f(q_h, q_f) = p(q_h + q_f)q_f - c_f q_f.$$  

We assume that the demand function is “not too convex” so that the profit of each firm is concave in its own quantity and that the marginal revenue is decreasing in the quantities of other firms (i.e., the quantities are strategic substitutes). This assumption guarantees the existence of the unique equilibrium. We use $\pi^*_h(c_f)$ and $\pi^*_f(c_f)$ to denote the equilibrium profits.

With probability $\theta$, the tariff $t$ is imposed on the export of the foreign firm. The objective function of the home firm is the same as (1). The objective function of the foreign firm is

$$p(q_h + q_f)q_f - \min \{c_f + t, c_s\} q_f$$

if it made investment, and

$$p(q_h + q_f)q_f - (c_f + t)q_f$$
if it did not make investment. The equilibrium profits are denoted by $\pi^*_h(c_f + t)$ and $\pi^*_f(c_f + t)$ if the foreign firm exports, and denoted by $\pi^*_h(c_s)$ and $\pi^*_f(c_s)$ if the foreign firm uses its subsidiary.

The assumptions of the strategic substitutes and the concavity of the objective function give the standard comparative statics results such that the equilibrium profit of each firm is decreasing in one’s own marginal cost and increasing in rival’s marginal cost. Namely, $\pi^*_h(\cdot)$ is increasing in its argument, and $\pi^*_f(\cdot)$ is decreasing in its argument.

### 3.2 Stage 2: FDI decision of the foreign firm

At stage 2, given the probability of protection, $\theta$, the foreign firm decides whether to make FDI or not. If the foreign firm makes FDI, its expected profit is

$$\theta \max \{ \pi^*_f(c_f + t), \pi^*_f(c_s) \} + (1 - \theta)\pi^*_f(c_f) - k,$$

where $k > 0$ is a fixed cost of the investment. If it does not make FDI, its expected profit is

$$\theta \pi^*_f(c_f + t) + (1 - \theta)\pi^*_f(c_f).$$

Obviously, the foreign firm does not make FDI when the tariff is small such that $c_f + t \leq c_s$ (i.e., the subsidiary production is more expensive than export). When $c_f + t > c_s$, the foreign firm will make FDI for the anticipatory tariff-jumping purpose if the gain of FDI is larger than the cost of FDI. That is, the foreign firm will make FDI if $\theta[\pi^*_f(c_s) - \pi^*_f(c_f + t)] > k$.

Now, define the cut-off level of probability above which the foreign firm will make FDI as $\theta_k$. Namely,

$$\theta_k = \frac{k}{\pi^*_f(c_s) - \pi^*_f(c_f + t)}$$

if $c_f + t > c_s$. The cut-off probability is increasing in $k$ and $c_s$, and decreasing in $t$. In words, the foreign firm is less likely to invest when the fixed cost of investment is higher or the marginal cost of subsidiary production is higher, and more likely to invest if the tariff is higher. For the convenience, let $\theta_k = 1$ if $c_f + t \leq c_s$. Then, the foreign firm’s FDI decision is described as follows\(^6\): it will make FDI if $\theta > \theta_k$, and will not make FDI if $\theta \leq \theta_k$.

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\(^6\)For the convenience of the further analysis, I assume that the FDI does not take place if the foreign firm is indifferent between investing and not investing.
### 3.3 Stage 1: A petition for protection

At stage 1, given the tariff the home firm decides how much to spend for a petition for protection. As we stated earlier, we assume that the probability of protection enforcement at stage 3 depends solely on the spending for a petition, and the probability is an increasing function of the spending. Thus, we model that the home firm is effectively choosing the probability of protection to maximize its expected profit, with the cost function $Z(\theta)$. That is, $Z(\theta)$ measures the cost of the protection-seeking effort by the home firm to obtain the probability of protection $\theta$. We assume that $Z(\theta)$ is increasing and strictly convex in $\theta$, reflecting the diminishing marginal return for the protection-seeking effort.

Because of the FDI opportunity of the foreign firm, the expected profit of the home firm, as a function of $\theta$, may consist of two parts. For $\theta \leq \theta_k$, the foreign firm will not make FDI, thus the expected profit of the home firm is given by

$$\theta \pi^*_h(cf + t) + (1 - \theta) \pi^*_h(cf) - Z(\theta).$$  
(2)

On the other hand, for $\theta > \theta_k$, the foreign firm will make FDI, thus the expected profit of the home firm is given by

$$\theta \pi^*_h(cs) + (1 - \theta) \pi^*_h(cf) - Z(\theta).$$  
(3)

Let $\theta_t$ and $\theta_s$ denote the probability that maximize (2) and (3) respectively. That is, $\theta_t$ satisfies the first-order condition

$$\pi^*_h(cf + t) - \pi^*_h(cf) - Z'(\theta_t) = 0,$$

and $\theta_s$ satisfies the first-order condition

$$\pi^*_h(cs) - \theta \pi^*_h(cf) - Z'(\theta_s) = 0.$$

Also, define

$$\Pi_h(\theta_t) \equiv \theta_t \pi^*_h(cf + t) + (1 - \theta_t) \pi^*_h(cf) - Z(\theta_t)$$

$$\Pi_h(\theta_k) \equiv \theta_k \pi^*_h(cf + t) + (1 - \theta_k) \pi^*_h(cf) - Z(\theta_k)$$

$$\Pi_h(\theta_s) \equiv \theta_s \pi^*_h(cs) + (1 - \theta_s) \pi^*_h(cf) - Z(\theta_s)$$

The optimal probability of protection chosen by the home firm, denoted by $\theta^*$, is given as follows:

$$\theta^* = \begin{cases} 
\theta_t & \text{if } \theta_t \leq \theta_k \\
\theta_k & \text{if } \theta_k \leq \theta_t < \theta_s \text{ or if } \theta_k < \theta_s \text{ and } \Pi_h(\theta_k) \geq \Pi_h(\theta_s) \\
\theta_s & \text{if } \theta_k < \theta_s \text{ and } \Pi_h(\theta_k) < \Pi_h(\theta_s)
\end{cases}$$  
(4)

\( \text{The second-order condition is satisfied since } Z(\theta) \text{ is convex.} \)
Although this may look complicated, the intuition explained below is quite straightforward.

First, consider $\theta_k = 1$. In this case, (3) is irrelevant since the foreign firm will never make FDI. Therefore, the home firm chooses $\theta_t$.

Now, consider $\theta_k < 1$. Notice that $\theta_k < 1$ implies $c_f + t > c_s$, which in turn implies $\pi_h^*(c_f + t) > \pi_h^*(c_s)$. Thus, for any given $\theta > 0$, (2) is larger than (3). This means that, at $\theta_k$ the expected profit of the home firm jumps down from (2) to (3). However, as long as $\theta_k$ is large enough that $\theta_t \leq \theta_k$, the FDI opportunity of the foreign firm does not matter to the home firm, since choosing $\theta_t$ does not cause the FDI of the foreign firm. That is, FDI of the foreign firm is “blockaded” in this case: the home firm chooses the probability of protection as if the foreign firm had no FDI opportunity, and at the chosen probability, the foreign firm will not make FDI. See Figure 1 for the illustration of this case.

If $\theta_t > \theta_k$, however, the FDI opportunity of the foreign firm matters to the protection-seeking decision of the home firm. Were the home firm to choose $\theta_t$, the foreign firm would make FDI, thus $\theta_t$ would be no longer optimal. So, instead of $\theta_t$, the home should choose the probability that “deters” the foreign firm from making FDI. Indeed, when $\theta_s \leq \theta_k < \theta_t$, the optimal probability for the home firm is $\theta_k$, the one that is just short of inducing FDI. We depict this case in Figure 2.

Finally, if $\theta_k$ is small enough that $\theta_k < \theta_s$, the home firm may want to allow, or “accommodate”, the foreign firm to make FDI. The FDI is still deterred if $\Pi_h(\theta_k) \geq \Pi_h(\theta_s)$, that is, the expected profit of deterring FDI is larger than the expected profit of accommodating FDI. On the other hand, the FDI is accommodated if $\Pi_h(\theta_k) < \Pi_h(\theta_s)$. See Figure 3 and 4 for illustration of these cases.

One notable feature of the optimal choice of $\theta$ is that the home firm may find it optimal to deter FDI. If the foreign firm had no FDI opportunity, the home firm would choose $\theta_t$. However, because of the FDI opportunity of the foreign firm, the home firm may choose $\theta_k$, which is less than $\theta_t$. In this sense, we claim that the mere existence of FDI opportunity, not the actual investment, can curb the protection-seeking effort of the home firm.

From (4) and the explanation given above, we have a rough idea about the transition pattern of $\theta^*$ in terms of $\theta_k$: the home firm chooses $\theta_t$ when $\theta_k$ is very large; as $\theta_k$ becomes smaller, the optimal choice is switched from $\theta_t$ to $\theta_k$; when $\theta_k$ becomes very small, the home firm may choose $\theta_s$. Recalling that $\theta_k$ is decreasing in $t$, we can describe this transition pattern of $\theta^*$ in terms of $t$. First, when $t$ is very small, $\theta_k$ is larger than $\theta_t$ thus the home firm chooses $\theta_t$. Then, as $t$ rises, eventually $\theta_k$ falls below $\theta_t$. Let $t$ denote
Figure 1:

Figure 2: The optimal choice of the probability: when $\theta_k$ is chosen (i)
Figure 3: The optimal choice of the probability: when $\theta_k$ is chosen (ii)

Figure 4: The optimal choice of the probability: when $\theta_s$ is chosen
$t$ such that $\theta_t(t) = \theta_k(t)$. At $\hat{t}$, the optimal choice of the probability of protection is switched from $\theta_t$ to $\theta_k$. For $t > \hat{t}$, the optimal choice is given by $\theta_k$. Finally, as $t$ becomes large, $\theta_k$ falls below $\theta_*$. Then, the home firm may choose $\theta_*$ instead of $\theta_k$.  

We depict this transition pattern of $\theta^*$ in Figure 5. Notice that from the definition of $\theta_t$, it is derived that $\partial \theta_t / \partial t > 0$ as long as $0 < \theta_t < 1$. Thus, when $\theta^* = \theta_t$, the optimal choice of the probability is increasing in $t$. In other words, as $t$ rises, the home firm increases its protection-seeking effort to increase the probability of protection. This is because a rise in $t$ increases $\pi_t^h(c_f + t)$, the gain from protection. On the other hand, when $\theta^* = \theta_k$, the optimal choice of the probability is decreasing in $t$. This is because, as $t$ rises, the foreign firm is going to make FDI at a smaller probability. To keep the foreign firm from making FDI, the home firm has to lower its protection-seeking effort.

Using Figure 5, we can see how the other parameters affect the choice of $\theta$. Recall that $\partial \theta_k / \partial k > 0$ and $\partial \theta_k / \partial c_s > 0$. When $k$ or $c_s$ decreases, $\theta_k$ curve shifts down in Figure 5. On the other hand, $\theta_t$ is unaffected by a decrease in $k$ or $c_s$. Therefore, as $k$ or $c_s$ decreases, $\hat{t}$ falls. This means that the optimal choice of the probability is switched from $\theta_t$ to $\theta_k$ at the smaller level of the tariff. The intuition is simple: when $k$ or $c_s$ is small, the foreign firm is going to make FDI at a smaller probability of protection, thus the optimal choice of the home firm is more likely to deter FDI.

Once the optimal level of protection probability has been chosen, the expected profit of the foreign firm, denoted by $\Pi_f(\theta^*)$, is given as follows.

$$
\Pi_f(\theta^*) = \begin{cases} 
\Pi_f(\theta_t) = \theta_t \pi^f_t(c_f + t) + (1 - \theta_t) \pi^s_t(c_f) & \text{if } \theta^* = \theta_t, \\
\Pi_f(\theta_k) = \theta_k \pi^f_t(c_f + t) + (1 - \theta_k) \pi^s_t(c_f) & \text{if } \theta^* = \theta_k, \\
\Pi_f(\theta_s) = \theta_s \pi^f_t(c_s) + (1 - \theta_s) \pi^s_t(c_f) - k & \text{if } \theta^* = \theta_s.
\end{cases}
$$

In Proposition 1, we present the comparative statics for the expected profit of the foreign firm with respect to $t$. Also, see Figure 6.

**Proposition 1** The expected profit of the foreign firm is decreasing in $t$ when $\theta^* = \theta_t$, increasing in $t$ when $\theta^* = \theta_k$, and constant in $t$ when $\theta^* = \theta_s$.

**Proof.** When $\theta^* = \theta_t$, the derivative of the expected profit of the foreign firm with respect to $t$ is given by

$$
\frac{\partial \Pi_f(\theta_t)}{\partial t} = \theta_t \frac{\partial \pi^f_t(c_f + t)}{\partial t} - \frac{\partial \theta_t}{\partial t} \left[ \pi^s_t(c_f) - \pi^f_t(c_f + t) \right] < 0.
$$

\(^8\)Whether $\theta_*$ is chosen depends on the comparison of $\Pi_h(\theta_k)$ and $\Pi_h(\theta_*)$. Thus, even when $t$ is so large that $\theta_k$ is smaller than $\theta_*$, $\theta_*$ may not be chosen. We will see this in detail later.
θ shifts down when k or cₜ falls.

Figure 5: The optimal choice of the probability: comparative statics
When $\theta^* = \theta_k$, the derivative of the expected profit of the foreign firm with respect to $t$ is given by

$$\frac{\partial \Pi_f(\theta_k)}{\partial t} = \theta_k \frac{\partial \pi_f^*(c_f + t)}{\partial t} - \frac{\partial \theta_k}{\partial t} \left[ \frac{\pi_f^*(c_f) - \pi_f^*(c_f + t)}{\pi_f^*(c_f)} \right] > 0,$$

because $\pi_f^*(c_f) > \pi_f^*(c_s) > \pi_f^*(c_f + t)$ when $\theta^* = \theta_k$.

When $\theta^* = \theta_s$, the expected profit of the foreign firm is independent of $t$ since $\theta_s$ is independent of $t$. 

If the foreign were not able to make FDI, the optimal probability of protection chosen by the home firm is always $\theta_t$, so that the foreign firm would be always hurt by an increase in the tariff. This is a conventional result. However, having the FDI opportunity after the protection-seeking of the home firm, the foreign firm can benefit from an increase in the tariff when the optimal choice of the home firm is to deter FDI. This is because, as the tariff increases, the foreign firm become more ready to make FDI at a smaller probability of protection. In order to keep the foreign firm from making FDI, the home firm has to choose smaller probability of protection. In turn, the decrease in the probability of protection increases the expected profit of the foreign firm.

Next, let us see the comparative statics for the expected profit of the home firm. When the optimal choice of the probability is $\theta_t$ or $\theta_s$, the comparative statics are simple. First, when $\theta^* = \theta_t$, the expected profit of the home firm is increasing in $t$, because $\partial \Pi_h(\theta_t)/\partial t = \theta_t [\partial \pi_h^*(c_f + t)/\partial t] > 0$ by the envelope theorem. Second, when $\theta^* = \theta_s$, the expected profit of the home firm is independent of $t$, thus a change in $t$ does not affect the profit. However, when $\theta^* = \theta_k$, whether the expected profit is increasing or decreasing in $t$ is not determinate, because

$$\frac{\partial \Pi_h(\theta_k)}{\partial t} = \theta_k \frac{\partial \pi_h^*(c_f + t)}{\partial t} + \frac{\partial \theta_k}{\partial t} \left[ \pi_h^*(c_f + t) - \pi_h^*(c_f) - Z'(\theta_k) \right] \geq 0. \quad (5)$$

The first term of equation (5) is positive, since a rise in the tariff increases the ex-post profit of the home firm. However, the second term of equation (5) is negative since $\partial \theta_k/\partial t < 0$ and $\pi_h^*(c_f + t) - \pi_h^*(c_f) - Z'(\theta_k) > 0$. That

---

$^9$The reason why $\pi_h(c_f + t) - \pi_h(c_f) - Z'(\theta_k) > 0$ is as follows. Note that $\pi_h(c_f + t) - \pi_h(c_f) - Z'(\theta_k)$ is the marginal expected profit of the home firm evaluated at $\theta_k$. The marginal expected profit is zero at $\theta_t$ by the first-order condition. Since $\theta_k < \theta_t$ when $\theta^* = \theta_k$, $\pi_h(c_f + t) - \pi_h(c_f) - Z'(\theta_k) > 0$ by the concavity of the expected profit.
Figure 6: The expected profit of the foreign firm.
is, an increase in the tariff lowers the expected profit through the decrease in the probability of protection. Therefore, in general, $\partial \Pi_h(\theta_k)/\partial t$ is not signable. The expected profit of the home firm can be decreasing in $t$ for some range of $t \in (\hat{t}, \bar{t})$. That is, the home firm may be hurt by an increase in the tariff.

Now, we consider under what conditions the expected profit of the home firm is always increasing in $t$. To simplify the analysis, suppose that the demand curve is linear, $p = a - b(q_h + q_f)$, and that the cost function of the protection-seeking effort is quadratic, $Z(\theta) = (z\theta^2)/2$ (thus $Z'(\theta) = z\theta$).

Then, the sign of $\partial \Pi_h/\partial t$ is characterized in terms of $c_s$, $k$, $z$ and $t$. In addition, analyzing the sign of $\partial \Pi_h/\partial t$ is useful to characterize under what conditions $\theta_s$ is chosen. Proposition 2 below present the results. Also, see Figure 7, 8, 9, 10, and 11.

**Proposition 2** Suppose that the demand curve is linear and the cost function of the protection-seeking effort is quadratic. Then,

1. for any given $c_s \in (c_f, c_f + \bar{t})$, there exists the corresponding value of $z_k$, denoted by $(z_k)_1$, above which $\partial \Pi_h/\partial t > 0$ for the whole range of $t \in [0, \bar{t}]$. If $z_k \geq (z_k)_1$, the home firm never chooses $\theta_s$.

2. for any given $c_s \in (c_f, c_f + \bar{t})$, there exists the corresponding value of $z_k$, denoted by $(z_k)_2$, below which the home firm chooses $\theta_s$ for some $t > \hat{t}$.

**Proof.** See Appendix 1.

Proposition 2-1 says that the home firm always gains from an increase in the tariff if $z$ and/or $k$ is large enough. This is understood by looking at equation (6) below, which is the same as equation (5), but in equation (6) we utilize the assumption of the quadratic cost function of the protection-seeking effort.

$$\frac{\partial \Pi_h(\theta_k)}{\partial t} = \theta_k \frac{\partial \pi_h^*(c_f + t)}{\partial t} + \frac{\partial \theta_k}{\partial t} \left[ \pi_h^*(c_f + t) - \pi_h^*(c_f) \right] - \frac{\partial \theta_k}{\partial t} \left[ \frac{z}{2} \left( \pi_f^*(c_s) - \pi_f^*(c_f + t) \right) \right]$$

A rise in the tariff affects the expected profit of the home firm through three ways. First, a rise in the tariff increases the ex-post profit (the first term in equation (6)). Second, the expected benefit of protection decreases due to the increase in the tariff, since $\theta_k$ is decreasing in $t$ (the second term in equation (6)). Third, however, the decrease in $\theta_k$ helps the home firm, since the cost of obtaining $\theta_k$ falls as $\theta_k$ decreases (the third term in equation (6)).
(6)). The size of \( z_k \) affects only the third term. The larger the \( z_k \) is, the larger the marginal cost of protection seeking, thus the more the home firm can save by a decrease in \( \theta_k \). Therefore, equation (6) is likely to be positive when \( z_k \) is large. Proposition 2-1 also says that \( \theta_s \) cannot be optimal if \( z_k \) is large enough that \( \Pi_h(\theta_k) \) is increasing in \( t \).

Consequently, for the home firm to choose FDI accommodation as the optimal choice, \( z_k \) must be small enough that \( \Pi_h(\theta_k) \) is decreasing in some \( t > \hat{t} \), and that \( \Pi_h(\theta_k) \) falls below \( \Pi_h(\theta_s) \). This is what Proposition 2-2 says. Its intuition is as follows. When the fixed cost of FDI is small, the probability of protection to deter FDI is small, thus deterring FDI becomes less preferable than accommodating FDI. When the marginal cost of protection seeking is small, the probability of protection to accommodate FDI is large, thus accommodating FDI becomes more preferable than deterring FDI.

In Figure 7, we numerically evaluate \((z_k)_1\) and \((z_k)_2\) for the case where the demand curve is \( p = 1 - q_h - q_f \), and the marginal costs of production are \( c_h = c_f = 0 \). In the diagram, we also show \((z_k)_0\) curve as a reference. In the area above \((z_k)_0\), the optimal choice of the probability is always blockading FDI (for derivation of \((z_k)_0\), see Appendix 2). By inspecting Figure 7, one can see that, for a large area of the parameter space, the home firm’s expected profit is always increasing in \( t \), thus FDI is not accommodated. This suggests that FDI does not occur very often when the home firm can influence the probability of protection in ADD protection.

In sum, the main implication of Proposition 1 and 2 is described as follows. When the tariff is small, the home firm chooses the probability of protection that blockades FDI. When FDI is blockaded, the home firm is benefitted from an increase in the tariff, and the foreign firm is hurt from an increase in the tariff. However, when the tariff is large enough, the home firm chooses the probability of protection that deters FDI. When the FDI is deterred, an increase in the tariff typically benefits both the home firm and the foreign firm.

### 3.4 Stage 0: Cournot competition

At stage 0, the home firm and the foreign firm play a Cournot game in the home country, without any protection of the home market and without any subsidiary production by the foreign firm. However, the tariff that may be enforced at stage 3 depends on the export of the foreign firm at stage 0. That is, \( t = t(q_f,0) \), where \( q_f,0 \) is the export of the foreign firm at stage 0. As we stated in the introduction, it is assumed that \( t'(q_f,0) > 0 \). In order to keep our presentation as simple as possible, here we focus on the cases
Figure 7: The sign of $\partial \Pi_h / \partial t$ and the transition pattern of $\theta^*$
Figure 8: The expected profit of the home firm: Region 0

Figure 9: The expected profit of the home firm: Region 1
Figure 10: The expected profit of the home firm: Region 2

Figure 11: The expected profit of the home firm: Region 3
where the parameters are in region 1 of Figure 7, thus the expected profit of the home firm is ever increasing in \( t \), and FDI accommodation does not occur for any \( t \).

For the home firm, its quantity at stage 0 has no intertemporal effect. Thus, it simply chooses the quantity of stage 0 to maximize the static profit, \( \pi_h(q_h, q_f) \) (hereafter, we do not use the time subscripts on \( q_h \) and \( q_f \) since the quantity variables we see in this subsection are those of stage 0. Also, we use the notation \( \pi_h(\cdot) \) and \( \pi_f(\cdot) \) to denote the profit of stage 1, and \( \pi_h^* (\cdot) \) and \( \pi_f^* (\cdot) \) to denote the profit of stage 3). The reaction function of the home firm at stage 0, denoted by \( r_h(q_f) \), is implicitly defined by

\[
\frac{\partial \pi_h(r_h(q_f), q_f)}{\partial q_h} = 0.
\]

On the other hand, the export of the foreign firm at stage 0 affects the tariff in stage 3. The foreign firm chooses its export to maximize the intertemporal profit

\[
V_f(q_h, q_f) \equiv \pi_f(q_h, q_f) + \delta \{ \theta_t(t(q_f)) \pi_f^*(c_f + t(q_f)) + [1 - \theta_t(t(q_f))] \pi_f^*(c_f) \},
\]

where \( \delta \in [0, 1) \) is a discount factor. We assume that \( V_f(q_h, q_f) \) is concave in \( q_f \) to satisfy the second-order condition, and that the marginal profit \( \partial V_f(q_h, q_f) / \partial q_f \) is decreasing in \( q_h \). These assumption guarantees that the slope of the reaction function is less than 1 in absolute value. Depending on whether the FDI is going to be blockaded (\( \theta^* = \theta_t \)) or deterred (\( \theta^* = \theta_k \)), the reaction function of the foreign firm is given by either \( r_f^t(q_h) \), which is defined by

\[
0 = \frac{\partial \pi_f(q_h, r_f^t)}{\partial q_f} + \delta \theta_t(t(r_f^t)) \frac{\partial \pi_f^*(c_f + t(r_f^t))}{\partial t} t'(r_f^t) - \delta \frac{\partial \theta_t(t(r_f^t))}{\partial t} [\pi_f^*(c_f) - \pi_f^*(c_f + t(r_f^t))] t'(r_f^t),
\]

or \( r_f^k(q_h) \), which is defined by

\[
\frac{\partial \pi_f(q_h, r_f^k)}{\partial q_f} + \delta \frac{\partial \theta_k(t(r_f^k))}{\partial t} [\pi_f^*(c_s) - \pi_f^*(c_f)] t'(r_f^k) = 0.
\]

Let \( r_f(q_h) \) denote the foreign firm’s reaction function in the static game. That is, \( r_f(q_h) \) is defined by

\[
\frac{\partial \pi_f(q_h, r_f(q_h))}{\partial q_f} = 0.
\]
By inspection, one can see that the sum of the second and the third terms of equation (7) is negative, while the second term of equation (8) is positive. Thus, from the concavity of the intertemporal profit function, it is derived that $r_f^t(q_h) < r_f(q_h) < r_f^k(q_h)$. On one hand, if the home firm is going to blockade FDI, then the foreign firm has an incentive to decrease its export from the static best-reaction level. On the other hand, if the home firm is going to deter FDI, the foreign firm has an incentive to increase its export from the static best-reaction level.

Those two different incentives to deviate from the static reaction function give the following two possible equilibria. One equilibrium occurs at the intersection of $r_h(q_f)$ and $r_f^t(q_h)$, denoted by $(q_{th}, q_{tf})$. In this equilibrium, the export of the foreign firm is below the static equilibrium quantity. Facing the future threat of protection (the petition for antidumping duty by the home firm), the foreign firm reduces its export, so as to lower the tariff and lower the protection seeking of the home firm. This is a conventional result in endogenous protection literature. The other equilibrium occurs at the intersection of $r_h(q_f)$ and $r_f^k(q_h)$, denoted by $(q_{kh}, q_{kf})$. In this equilibrium, it is interesting to see that the export of the foreign firm is above the static equilibrium quantity. In other words, the foreign firm “dumps” more in the prospect of antidumping policy. Facing the future threat of protection, the foreign firm chooses to export more to increase the tariff. By increasing the future tariff, the foreign firm makes itself ready to make FDI at a smaller level of protection probability. Then, the home firm, which wants to deter the FDI, will lower its protection seeking and lower the protection probability. We name this strategic increase in the export “protection-threat-defusing” export. Proposition 3 below formally gives under what conditions each equilibrium occurs. Also, see Figure 12, 13, and 14.

**Proposition 3** Let $\tilde{q}_h$ be the value of $q_h$ such that the foreign firm is indifferent between exporting $r_f^t(q_h)$ and exporting $r_f^k(q_h)$. That is, $V_f(\tilde{q}_h, r_f^t(\tilde{q}_h)) = V_f(\tilde{q}_h, r_f^k(\tilde{q}_h))$. Then

1. If $q_h^k > \tilde{q}_h$, $(q_{kh}, q_{kf}^k)$ is the unique equilibrium.
2. If $q_h^t > \tilde{q}_h$, $(q_{kh}, q_{tf}^k)$ is the unique equilibrium.
3. If $q_h^k \leq \tilde{q}_h \leq q_h^t$, both $(q_{kh}, q_{tf}^k)$ and $(q_{h}^k, q_{tf}^k)$ are the equilibria.

**Proof.** Since the slopes of the reaction functions are negative and less than 1 in absolute value, and since $r_f^t(q_h) < r_f^k(q_h)$, $q_h^k$ is less than $q_h^t$. Then, it suffices to show that the reaction function of the foreign firm is given by $r_f^t(q_h)$ when $q_h \geq \tilde{q}_h$, and it is given by $r_f^k(q_h)$ when $q_h \leq \tilde{q}_h$. Differentiating
Thus the larger $q_h$ is, the more profitable for the foreign firm to choose $r_f^t(q_h)$ rather than $r_f^k(q_h)$. Hence, the reaction function of the foreign firm is given by $r_f^t(q_h)$ when $q_h \geq \tilde{q}_h$, and given by $r_f^k(q_h)$ when $q_h \leq \tilde{q}_h$, as claimed.

Proposition 3 says that the protection-threat-defusing export, $q_f^k$, is the unique equilibrium quantity if $\tilde{q}_h$ is larger than $q_t^h$. In the diagram (see Figure 12, 13, and 14), this means that $q_f^k$ is more likely to be the unique equilibrium if the foreign firm has an incentive to increase its export above the static reaction quantity for the wider range of $q_h$. From the definition of $\tilde{q}_h$ and $q_t^h$, it is straightforward to show that $\partial \tilde{q}_h / \partial k < 0$ and $\partial \tilde{q}_h / \partial c_s < 0$ while $\partial q_f^k / \partial k = 0$ and $\partial q_f^k / \partial c_s = 0$. Thus, when the fixed cost of FDI or the marginal cost of subsidiary production is small enough, the protection-threat-defusing export occurs as the unique equilibrium. Intuitively, the smaller $k$ or the smaller $c_s$ is, the more likely the optimal probability of protection chosen by the home firm is to deter FDI, thus the more likely the foreign firm is to engage in protection-threat-defusing export for given $q_h$.

Whether the protection-threat-defusing export is the unique equilibrium also depends on the shape of the tariff function, $t(q_f)$. For example, suppose that $t(0) \geq \hat{t}$. Then, the probability of protection chosen by the home firm is to deter FDI no matter how much the foreign firm exports. Thus, in this extreme case, the foreign firm has no incentive to decrease its export, therefore the protection-threat-defusing export is the unique equilibrium. This suggests that the protection-threat-defusing export is likely to be the unique equilibrium if the home government is going to impose a high tariff even when $q_f$ is small.

Before concluding this section, it is worth mentioning that the protection-threat-defusing export by the foreign firm does not necessarily hurt the home firm. When the protection-threat-defusing export occurs in the equilibrium, the home firm’s profit in stage 0 is lower than the profit of the static game, since its equilibrium quantity is smaller than the static equilibrium quantity. However, in the later stage, the home firm gains from an increase in the tariff due to the increase in the export of the foreign firm at stage 0. Thus, the
Figure 12: Equilibrium in stage 0: \((q_h^f, q_f^f)\) is the unique equilibrium
Figure 13: Equilibrium in stage 0: \((q^k_h, q^k_f)\) is the unique equilibrium
Figure 14: Equilibrium in stage 0: both \((q^t_h, q^t_f)\) and \((q^k_h, q^k_f)\) are the equilibria
overall effect of the protection-threat-defusing export on the intertemporal profit of the home firm is ambiguous.

4 Concluding remarks

This paper analyzed how the protection-seeking effort of the import-competing firm, in the form of an antidumping duty petition, is affected by the FDI opportunity of the foreign firm after the protection seeking takes place. We have shown that the optimal level of protection-seeking effort chosen by the home firm is either blockading FDI, deterring FDI, or accommodating FDI. One interesting result derived in this paper is that when the optimal choice is to deter FDI, the home firm decreases its protection-seeking effort as the tariff increases. An implication of this inverse relationship between the tariff and the protection-seeking effort is that an increase in the tariff can benefit the foreign firm.

When the protection is endogenized in a way that the future tariff depends on the current export of the foreign firm, the foreign firm will strategically change its export in order to lower the protection-seeking effort of the home firm. The second point of the paper is that the direction of the strategic change in the export of the foreign firm is not uniform. If the cost of investment is high, FDI is likely to be blockaded, so that the foreign firm strategically decrease its export to lower the protection-seeking effort of the home firm. On the other hand, if the cost of investment is low enough, FDI is likely to be deterred, so that the foreign firm strategically increase its export to lower the protection-seeking effort of the home firm. The results are similar to the ones derived in Blonigen and Ohno (1998), but the empirical implications are quite different. In our model, the strategic change in the export may not be followed by an actual investment in the later period. Our paper suggests that, often times the mere existence of the investment opportunity is enough to influence the protection-seeking decision of the home firm. The strategic purpose of the foreign firm to increase its export is, thus, to make the threat of investment more credible.
Appendix 1: Proof of proposition 2

Proof of proposition 2-1

When $Z(\theta) = (z\theta^2)/2$, equation (5) can be rewritten as

$$
\frac{\partial \Pi_h(\theta_k)}{\partial t} = \theta_k \frac{\partial \pi^*_h(c_f + t)}{\partial t} + \frac{\partial \theta_k}{\partial t} [\pi^*_h(c_f + t) - \pi^*_h(c_f) - z\theta_k] \quad (A1)
$$

and $X(t, c_s) = [\pi^*_f(c_s) - \pi^*_f(c_f + \hat{t})]^2 \frac{\partial \pi^*_h(c_f + t)}{\partial t} + [\pi^*_f(c_s) - \pi^*_f(c_f + \hat{t})][\pi^*_h(c_f + t) - \pi^*_h(c_f)]$.

Since $A(t, c_s) < 0$, it can be seen that $\frac{\partial \Pi_h(\theta_k)}{\partial t} \geq 0$ for the range of $t \in [\hat{t}, \bar{t}]$ if

$$
zk \geq \max_{t \in [\hat{t}, \bar{t}]} X(t, c_s).
$$

It is straightforward to show that $\frac{\partial X}{\partial t}|_{t = c_s - c_f} > 0$, $\lim_{t \to \bar{t}} \frac{\partial X}{\partial t} = 0$, and $X(t, c_s)$ is concave for $t \in [c_s - c_f, \bar{t}]$ if the demand curve is linear. Hence, $X(t, c_s)$ has the unique maximum at some $t \in [c_s - c_f, \bar{t}]$. Letting $t_X$ denote the argmax of $X(t, c_s)$, therefore, $(zk)_1$ is defined by

$$(zk)_1 \equiv X(t_X(c_s), c_s).$$

Because $\frac{\partial X}{\partial t}|_{t = c_s - c_f} > 0$ and $X(t, c_s) = 0$ at $t = c_s - c_f$, we have $(zk)_1 > 0$.

For this definition of $(zk)_1$ to be valid, it needs to be confirmed that $t_X \geq \hat{t}$ when $zk = (zk)_1$ (notice that $\hat{t}$ depends on $zk$). Since $\theta_t = [\pi^*_h(c_f + t) - \pi^*_h(c_f)]/z$ and $\theta_k = k/[\pi^*_f(c_s) - \pi^*_f(c_f + \hat{t})]$, when $zk = (zk)_1$, $\hat{t}$ is given by

$$
[\pi^*_h(c_f + \hat{t}) - \pi^*_h(c_f)][\pi^*_f(c_s) - \pi^*_f(c_f + \hat{t})] = (zk)_1.
$$

The following chain of inequalities

$$
[\pi^*_h(c_f + \hat{t}) - \pi^*_h(c_f)][\pi^*_f(c_s) - \pi^*_f(c_f + \hat{t})] = (zk)_1 = X(t_X(c_s), c_s) < [\pi^*_h(c_f + t_X(c_s)) - \pi^*_h(c_f)][\pi^*_f(c_s) - \pi^*_f(c_f + t_X(c_s))]
$$

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confirms that $t_X \geq \hat{t}$ when $z_k = (z_k)_1$, since $[\pi^*_h(c_f + t) - \pi^*_h(c_f)][\pi^*_f(c_s) - \pi^*_f(c_f + t)]$ is increasing in $t$.

At $t = \hat{t}$, $\Pi_h(\theta_k) = \Pi_h(\theta_t) > \Pi_k(\theta_s)$. Since $\partial \Pi_f(\theta_k)/\partial t > 0$ for $t \in [\hat{t}, t]$ if $z_k > (z_k)_1$, this suffices to prove the second argument of proposition 8-1.

**Proof of proposition 2-2.**

For some $t > \hat{t}$, the home firm chooses $\theta_s$ if

$$\Pi_h(\theta_s) > \min_{t \in [\hat{t}, t]} \Pi_h(\theta_k(t), t).$$

(A2)

From equation (A1), it is seen that $\partial \Pi_h(\theta_k)/\partial t = 0$ when $X(t, c_s) = z_k$. Since $X(t, c_s)$ is single-peaked in $t \in [\hat{t}, \hat{t}]$, there are at most two $t$'s that satisfy $X(t, c_s) = z_k$. Let $t^+_0$ and $t^-_0$, with $t^+_0 \geq t^-_0$, denote such $t$'s. Since $t^+_0 \geq t_X \geq t^-_0$, $\partial^2 \Pi_h(\theta_k)/\partial t^2 \leq 0$ at $t = t^-_0$, and $\partial^2 \Pi_h(\theta_k)/\partial t^2 \geq 0$ at $t = t^+_0$. Thus, For $t \in [\hat{t}, \hat{t}]$, there is the unique minimum of $\Pi_h(\theta_k)$ at $t = t^+_0$. Noticing that $\theta_s = [\pi^*_h(c_s) - \pi^*_h(c_f)]/z$, and evaluating $\Pi_h(\theta_k(t), t)$ at $t^+_0$, the equation (A2) is rewritten as

$$\frac{[\pi^*_h(c_s) - \pi^*_h(c_f)]^2}{2\pi^*_f(c_s) - \pi^*_f(c_f + t^+_0)} > z_k \left\{ [\pi^*_h(c_f + t^-_0) - \pi^*_h(c_f)] - \frac{z_k}{2[\pi^*_f(c_s) - \pi^*_f(c_f + t^+_0)]} \right\}.$$  \hspace{1cm} (A3)

$(z_k)_2$ is found by setting (A3) in equality and solving for $z_k$. For the time being, taking $t^+_0$ as given, the solutions of the equality of (A3) are

$$z_k = \frac{[\pi^*_h(c_f + t^+_0) - \pi^*_h(c_f)][\pi^*_f(c_s) - \pi^*_f(c_f + t^+_0)]}{\left[\pi^*_h(c_f + t^-_0) - \pi^*_h(c_f)\right]^2 - [\pi^*_h(c_s) - \pi^*_h(c_f)]^2},$$  \hspace{1cm} (A4)

and

$$z_k = \frac{[\pi^*_h(c_f + t^-_0) - \pi^*_h(c_f)][\pi^*_f(c_s) - \pi^*_f(c_f + t^-_0)]}{\left[\pi^*_h(c_f + t^-_0) - \pi^*_h(c_f)\right]^2 - [\pi^*_h(c_s) - \pi^*_h(c_f)]^2}.$$  \hspace{1cm} (A5)
(A4) and (A5) are the candidates of \((zk)_2\). Suppose that (A4) defines \((zk)_2\). Then, because \(t_0^* \geq t_X\),

\[
(zk)_2 = [\pi_h^*(cf + t_0^*) - \pi_h^*(cf)] [\pi_f^*(cs) - \pi_f^*(cf + t_0^*)] + \text{a positive term}
\]

\[
> [\pi_h^*(cf + t_0^*) - \pi_h^*(cf)] [\pi_f^*(cs) - \pi_f^*(cf + t_0^*)]
\]

\[
\geq [\pi_h^*(cf + t_X) - \pi_h^*(cf)] [\pi_f^*(cs) - \pi_f^*(cf + t_X)]
\]

\[
> (zk)_1.
\]

This is a contradiction, since \((zk)_2\) should be less than \((zk)_1\). So, it is (A5) that is relevant to define \((zk)_2\). Now, noticing that \(t_0^*\) is a function of \(zk\), \((zk)_2\) is implicitly defined by

\[
(zk)_2 \equiv \frac{[\pi_h^*(cf + t_0^*((zk)_2)) - \pi_h^*(cf)] [\pi_f^*(cs) - \pi_f^*(cf + t_0^*((zk)_2))] - [\pi_f^*(cs) - \pi_f^*(cf + t_0^*((zk)_2))] \times \sqrt{[\pi_h^*(cf + t_0^*((zk)_2)) - \pi_h^*(cf)]^2 - [\pi_h^*(cs) - \pi_h^*(cf)]^2}.
\]

Since the right hand side of the equation is positive, it is readily seen that \((zk)_2 > 0\).

**Appendix 2: Derivation of \((zk)_0\)**

The optimal choice of the probability is given by \(\theta_t\) for the whole range of \(t \in [0, \bar{t}]\), if \(\theta_t(\bar{t}) \leq \theta_k(\bar{t})\). So, \((zk)_0\) curve, above which the optimal choice is always \(\theta_t\) is defined by \(\theta_t(\bar{t}) = \theta_k(\bar{t})\). Rearranging this equality, we obtain

\[
(zk)_0 = [\pi_h^*(cf + \bar{t}) - \pi_h^*(cf)] \pi_f^*(cs).
\]
References


